Evaluate the following integrals, or explain why they can't be evaluated. **SCORE: _____ / 55 PTS**

[a] $\int (4r-7)(3-2r)^9\,dr$

④ $u = 3-2r \longrightarrow r = \frac{3-u}{2}$

$du = -2\,dr$

$dr = -\frac{1}{2}\,du$

$(4r-7)(3-2r)^9\,dr$

$= -\frac{1}{2}\left(4\left(\frac{3-u}{2}\right)-7\right)u^9\,du$

$= -\frac{1}{2}(-2u-1)u^9\,du$

$= \frac{1}{2}(2u^{10}+u^9)\,du$

$\int \frac{1}{2}(2u^{10}+u^9)\,du$

$= \frac{1}{11}u^{11} + \frac{1}{20}u^{10} + C$

$= \frac{1}{11}(3-2r)^{11} + \frac{1}{20}(3-2r)^{10} + C$ ①

[b] $\int_{-2}^{2} \frac{7x-4x^3}{x^4-7x^2+10}\,dx$

$x^4-7x^2+10 = (x^2-2)(x^2-5) = 0$

@ $x = \pm\sqrt{2}, \pm\sqrt{5}$

③ $\in [-2,2]$

INTEGRAND IS DISCONTINUOUS,

SO FTC DOESN'T APPLY

② EACH

EXCEPT AS NOTED

[c] $\int \frac{15\csc^2 4t - 10t}{\cos^2(4t^2+3\cot 4t)}\,dt$

④ $u = 4t^2 + 3\cot 4t$

$du = (8t - 12\csc^2 4t)\,dt$

$-\frac{5}{4}\,du = (-10t + 15\csc^2 4t)\,dt$

④ $\int -\frac{5}{4}\frac{1}{\cos^2 u}\,du$

$= -\frac{5}{4}\int \sec^2 u\,du$

$= -\frac{5}{4}\tan u + C$ ③

$= -\frac{5}{4}\tan(4t^2+3\cot 4t) + C$ ①

[d] $\int_{-3}^{3} \left(y\sqrt{81-y^4} - 2\sqrt{36-(y+3)^2}\right)dy$

$= \int_{-3}^{3} y\sqrt{81-y^4}\,dy - 2\int_{-3}^{3}\sqrt{36-(y+3)^2}\,dy$

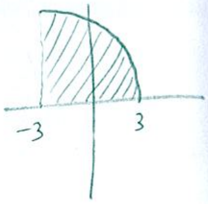
③ $(-y)\sqrt{81-(-y)^4}$

$= -y\sqrt{81-y^4}$

INTEGRAND IS ODD,

CONTINUOUS

INTEGRAL = 0



$= -2\cdot\frac{1}{4}\pi(6)^2$ ④

$= -18\pi$

Find $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{19i}{\left(\sqrt[3]{8+\frac{19i}{n}}\right)n}$ by finding the corresponding definite integral, and evaluating that integral. **SCORE: _____ / 10 PTS**

$$\underbrace{\int_{8}^{27}}_{}\underbrace{\frac{1}{\sqrt[3]{x}}}_{}\,dx = \int_{8}^{27} x^{-\frac{1}{3}}\,dx = \underbrace{\frac{3}{2}x^{\frac{2}{3}}}_{}\Big|_{8}^{27} = \frac{3}{2}\left(27^{\frac{2}{3}} - 8^{\frac{2}{3}}\right) = \frac{3}{2}(9-4) = \underbrace{\frac{15}{2}}_{}$$

② EACH

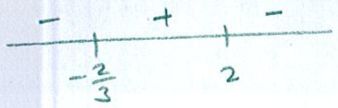
Using proper English and mathematical notation, state both parts of the Fundamental Theorem of Calculus, as well as the Net Change Theorem. **SCORE: _____ / 15 PTS**

The acceleration of an object at time t (in seconds) is given by $a(t) = 4 - 6t$ (in meters per second^2). **SCORE: _____ / 25 PTS**
The initial velocity of the object is 4 meters per second. Find the distance travelled by the object from $t = 1$ to $t = 6$.

$$v(t) = \int a(t)\,dt = \underbrace{\int (4-6t)\,dt}_{} \overset{③}{=} \underbrace{4t - 3t^2 + C}_{} \ ③$$

$$v(0) = \underbrace{4 = C}_{} \ ③$$

$$v(t) = 4 + 4t - 3t^2 = (2-t)(2+3t)$$



$$\int_{1}^{6} |4 + 4t - 3t^2|\,dt$$

$$= \underbrace{\int_{1}^{2} (4 + 4t - 3t^2)\,dt}_{} \overset{④}{} + \underbrace{\int_{2}^{6} -(4 + 4t - 3t^2)\,dt}_{} \overset{④}{}$$

$$= \underbrace{(4t + 2t^2 - t^3)\Big|_{1}^{2} + (-4t - 2t^2 + t^3)\Big|_{2}^{6}}_{} \ ④$$

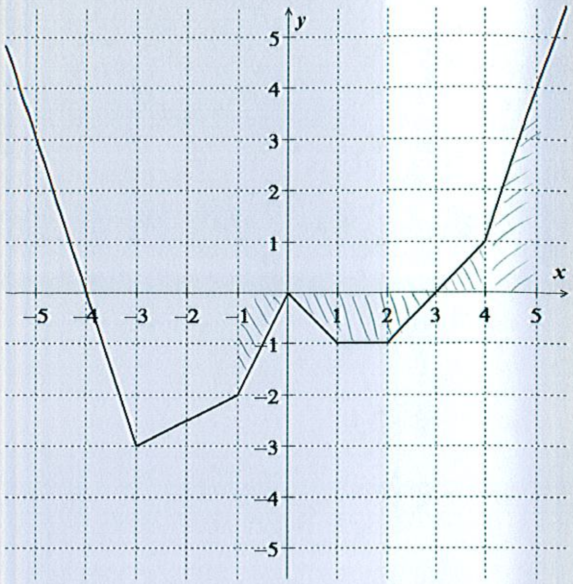
$$= (8 \cancel{+8} \cancel{-8}) - (4 + 2 - 1) + (-24 - 72 + 216) - (-8 \cancel{-8} \cancel{+8})$$

$$= 8 - 5 + 120 + 8$$

$$= \underbrace{131}_{} \ \underbrace{\text{METERS}}_{} \ ①$$

③

Let $g(x) = \int_5^x f(t)\,dt$, where f is the function whose graph is shown on the right. **SCORE: _____ / 30 PTS**



[a] Find $g(-1)$.

$$\int_5^{-1} f(t)\,dt = \underbrace{-\int_{-1}^{5} f(t)\,dt}_{\textcircled{2}}$$

$$= -\Big[\underbrace{-\tfrac12(1)(2)}_{\textcircled{1\frac12}} \underbrace{-\tfrac12(1)(3+1)}_{\textcircled{1\frac12}} \underbrace{+\tfrac12(1)(1)}_{\textcircled{1\frac12}} \underbrace{+\tfrac12(1)(1+4)}_{\textcircled{1\frac12}}\Big]$$

$$= -\left(-1-2+\tfrac12+\tfrac52\right) = \underbrace{0}_{\textcircled{1}}$$

[b] Find $g'(2)$. **Explain your answer very briefly.**

$$\underbrace{g'(2) = f(2)}_{\textcircled{3}} = \underbrace{-1}_{\textcircled{2}}$$

[c] Find all critical numbers of g. **Explain your answer very briefly.**

$$\underbrace{g'(x) = f(x) = 0}_{\textcircled{4}} \text{ AT } x = \underbrace{-4, 0, 3}_{\textcircled{3}}$$

[d] Find the $x-$coordinates of all local maxima of g. **Explain your answer very briefly.**

$$\underbrace{g'(x) = f(x) = 0}_{\textcircled{3}} \text{ AND } \underbrace{\text{CHANGES FROM} > 0 \text{ TO} < 0}_{\textcircled{4}}$$

AT $x = \underbrace{-4}_{\textcircled{2}}$

If f is continuous and $\int_{-1}^{5} f(t)\,dt = 9$, find $\int_{-1}^{2} (8 - 6f(3-2t))\,dt$. **SCORE: _____ / 15 PTS**

② EACH
EXCEPT AS NOTED

$$\underbrace{\int_{-1}^{2} 8\,dt - 6\int_{-1}^{2} f(3-2t)\,dt}$$

↓ ↓

$8(2 - -1)$ $-6(\frac92)$

$= 24$ $= -27$

$= \underbrace{24}\underbrace{-27}$

$= \underbrace{-3}_{\textcircled{1}}$

$$\int_{-1}^{2} f(3-2t)\,dt$$

$\underline{u = 3-2t}$ $<$ $t=2 \rightarrow u=-1$; $t=-1 \rightarrow u=5$

$du = -2\,dt$

$-\frac12\,du = dt$

$$\int_{5}^{-1} \underbrace{-\tfrac12 f(u)\,du} = \tfrac12 \int_{-1}^{5} f(u)\,du$$

$= \frac12(9) = \frac92$